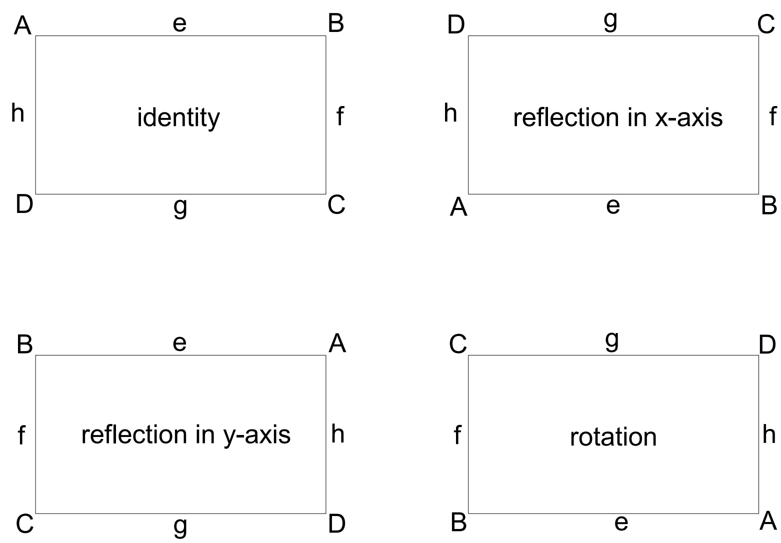
Math 522 Exam 4 Solutions

1. Prove the converse of Wilson's theorem. That is, suppose that $p > 1$ is not prime. Prove that $p \nmid (p-1)! + 1$.

   Since p is not prime, it is not irreducible, so we may write $p = mn$ where $1 < m < p$. Because $m \leq p - 1$, we have $m|(p-1)!$. Suppose that also $p|(p-1)! + 1$; but then $m|(p-1)! + 1$ since $m|p$. Then we would have $m|\left((p-1)! + 1\right) - (p-1)!$ so $m = 1$, a contradiction.

2. A Vadim diagram is a (non-square) rectangle, with each edge and each corner colored black or white. How many "different" Vadim diagrams are there? (rotations and reflections are not considered different)



   The above are how the four group elements act on our rectangle. We first show which vertices must have the same color, and which edges; then, we show the number of Vadim diagrams invariant under that particular group element action.

   identity: all 8 are independently chosen, $2^8 = 256$
   reflection in x-axis: A=D, B=C, e=g, $2^5 = 32$
   refleciton in y=axis: A=B, C=D, f=h, $2^5 = 32$
   rotation: A=C, B=D, e=g, f=h, $2^4 = 16$

   Hence, the total number of different Vadim diagrams is
   $\frac{1}{4}(256 + 32 + 32 + 16) = 84$.